Exam. Code : 211002
Subject Code : 4274

M.Sc. (Mathematics) 2nd Semester

REAL ANALYSIS—II

Paper—MATH-561

Time Allowed—Three Hours] [Maximum Marks—100

Note :— Candidates are required to attempt **FIVE** questions selecting at least **ONE** question from each section. The fifth question may be attempted from any section.

SECTION—A

1. Suppose $f_n \to f$ uniformly on a set E in metric space. Let x be a limit point of E and $\lim_{t \to x} f_n(t) = A_n$. Then prove that $\{A_n\}$ converges and $\lim_{t \to x} f(t) = \lim_{n \to \infty} A_n$. 20

2. State and prove Stone-Weierstrass theorem. 20

SECTION—B

3. (a) Prove that Lebesgue outer measure is translation invariant. 10

   (b) Prove that a set E is measurable if and only if there is a G_δ set G with $E \subset G$ and $m^*(G \backslash E) = 0$. 10

4. (a) Construct a non-measurable set. 12

   (b) Show that a measurable function is almost a continuous function. 8

SECTION—C

5. (a) State and prove Littlewood's third principle. 10

(b) Let u_n be a sequence of non-negative measurable functions and let $f = \sum_{n=1}^{\infty} u_n$. Then prove that

$$\int f = \sum_{n=1}^{\infty} \int u_n .$$ 10

6. (a) State and prove Lebesgue Convergence Theorem. 10

(b) Let f be a bounded Riemann integrable function on [a, b]. Prove that f is Lebesgue integrable. Is the converse true ? Justify. 10

SECTION—D

7. (a) Let f be absolutely continuous function on [a, b] and $f'(x) = 0$ a.e. Then prove that f is constant. 10

(b) State and prove Vitali's Lemma. 10

8. (a) Prove that a function F is an indefinite integral if and only if it is absolutely convergent. 10

(b) If f is integrable on [a, b] and $\int_a^x f(t)\,dt = 0 \; \forall \, x \in [a, b]$ then prove that $f = 0$ a.e. in [a, b]. 10